Geometry for Middle School Teachers

(sample pages from chapter 3)

Draft Copy

John Beem

Department of Mathematics
University of Missouri-Columbia

beem@math.missouri.edu

Making Wump Hats

Zack and Marta experimented with multiplying each of Mug's coordinates by different whole numbers to create other similar figures. Marta wondered how multiplying the coordinates by a decimal, or adding numbers to or subtracting numbers from each coordinate, would affect Mug's shape. When she asked her uncle about this, he gave her the coordinates for a new shape—a hat for Mug to wear—and some rules to try on the shape.

	Hat 1	Hat 2	Hat 3	Hat 4	Hat 5	Hat 6
Point	(x, y)	$(x + 2, y + 2)$	$(x + 3, y - 1)$	$(2x, y + 2)$	$(2x, 3y)$	$(0.5x, 0.5y)$
A	(0, 4)	(2, 6)	(3, 3)	(0, 6)	(0, 12)	(0, 2)
B	(0, 1)					
C	(6, 1)					
D	(4, 2)					
E	(4, 4)					
F	(3, 5)					
G	(1, 5)					
H	(0, 4)					

Problem 2.3

Use the table and dot paper grids on Labsheets 2.3A and 2.3B.

- To make Mug's hat, plot points *A*–*H* from the Hat 1 column on the grid labeled Hat 1, connecting the points as you go.
- For Hats 2–6, use the rules in the table to fill in the coordinates for each column. Then, plot each hat on the appropriate grid, connecting the points as you go.

Figure 3.4.3

Reproduced from page 21 of *Stretching and Shrinking* in the CMP grade 7 materials. Note that students are being given gentle introductions to coordinate geometry and to functions of two variables. Transformations of the form $F(x, y) = (ax + h, by + k)$ take each axis to itself or a line parallel to the axis.

One very simple way to obtain similar polygons with scale factor $\boldsymbol{S} = |k|$, is to take transformations of the very special form $F(x, y) = (kx, ky)$ for some $k \neq 0$. These transformation take each polygon P_1 in the x-y plane to a similar polygon P_2 and the scale factor is $|k|$. Also, these transformations take the origin $(0, 0)$ to itself. They are dilations with center at the origin. When the number k is greater than one, they take each point away from the origin in a radial manner with each point moved further away by the factor k. For example, if $k = 2$, then each point is moved directly away from the origin to the point twice as far from the origin as it was originally located. If $0 < k < 1$, then each point is moved in toward the origin along the segment joining the point to the origin. For example, if $k = 1/2$, then each point is move half the distance toward the origin. When k is negative, $F(x, y) = (kx, ky)$ may be thought of as taking (x, y) to $(|k|x, |k|y)$ followed by a reflection across the origin.

Exercises 3.4

1. Consider the transformation $F(x, y) = (2x, 2y)$. Let ΔABC have vertices A = $(1, 1)$, B = $(3, 4)$ and C = $(1, 6)$. Let A', B' and C' be the respective images under F of A, B, and C.
   (a) Find the coordinates of A', B', C'
   (b) Find the distances AB, AC, BC, A'B', A'C', and B'C'.
   (c) Use the SSS Similarity Theorem to prove that $\Delta ABC \sim \Delta A'B'C'$.

2. Let $F(x, y) = (kx, ky)$ for some $k \neq 0$. Prove that for all pairs of points (x_1, y_1) and (x_2, y_2), the Euclidean distance between the pair of image points $F(x_1, y_1)$ and $F(x_2, y_2)$ is $|k|$ times the Euclidean distance between the original pair (x_1, y_1) and (x_2, y_2).

3. An equilateral triangle has an altitude of height 12.
   (a) Find the length of a side of this triangle.
   (b) Find the perimeter of this triangle.
   (c) Find the area of this triangle.

4. An equilateral triangle has each side of length k.
   (a) Find the length of an altitude of this triangle in terms of k.
   (b) Find the perimeter of this triangle in terms of k.
   (c) Find the area of this triangle in terms of k.

5. Assume you are given two regular hexagons. One has each side of length 2 units and the other has each side of length 10 units.
   (a) Find the perimeter of each of these hexagons.
   (b) Find the area of each of these hexagons.

6. Assume you are given two regular hexagons. One has each side of length w and the other has each side of length $5w$.
   (a) Find the perimeter of each of these hexagons in terms of w.
   (b) Find the area of each of these hexagons in terms of w.

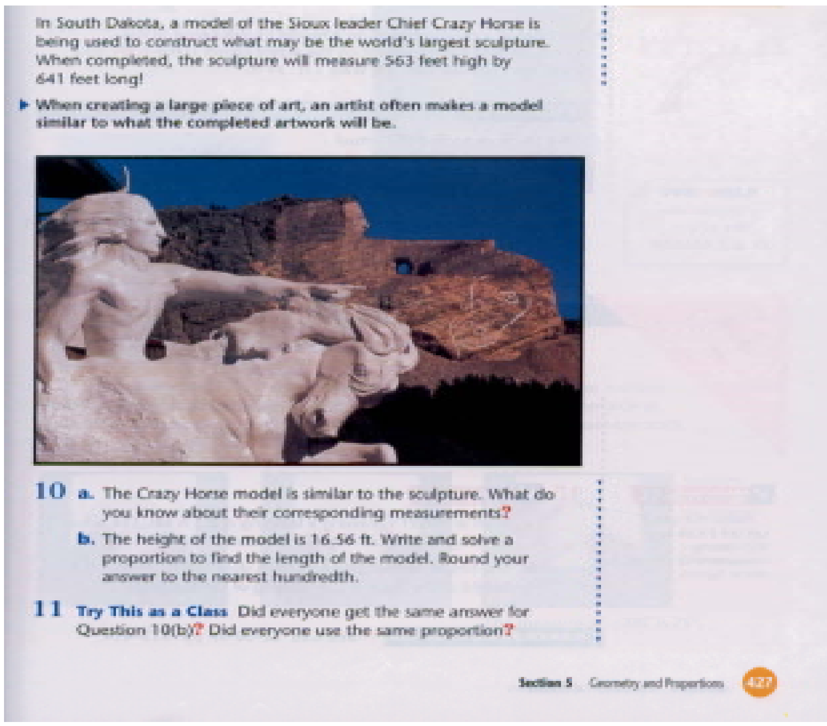
7. Let P_1 and P_2 be polygons such that $P_1 \sim P_2$ with scale factor $\boldsymbol{S}$. Assume that P_1 has a perimeter of 6,000 units and an area of 4,000 square units. Assume that P_2 has an area of 10 square units.
   (a) Find the scale factor $\boldsymbol{S}$.
   (b) Find the perimeter of P_2.

Exploration

- Divide the students in a class into 5 teams and have each team assigned one of the hats #2 to #6 in Figure 3.4.3. Each team should construct the figure for their hat and also find the area of the original hat #1 as well as the area for their hat. Each team should also find the ratio of the area for their hat to the area of the original hat. The class should be able to relate the ratio of areas for a given hat to the coefficients of x and y in the rule for constructing that hat. Students should be able to relate the product $a \bullet b$ to the effect F has on areas of figures and they should be able to guess how a and b should be related for the new figures to always be similar to the original figures.

3.5 Similarity for more general figures

In Section 3.1 similarity was defined for polygons. Clearly the idea of similarity goes beyond polygons to more general pairs of figures which have the same shape, but need not have the same size. This raises the question of how to define similarity for figures that fail to be polygons.



In South Dakota, a model of the Sioux leader Chief Crazy Horse is being used to construct what may be the world's largest sculpture. When completed, the sculpture will measure 563 feet high by 641 feet long!

▸ **When creating a large piece of art, an artist often makes a model similar to what the completed artwork will be.**

10 **a.** The Crazy Horse model is similar to the sculpture. What do you know about their corresponding measurements?

b. The height of the model is 16.56 ft. Write and solve a proportion to find the length of the model. Round your answer to the nearest hundredth.

11 **Try This as a Class** Did everyone get the same answer for Question 10(b)? Did everyone use the same proportion?

Section 5 Geometry and Proportions 427

Figure 3.5.1

Reproduced from page 427 of *Book 1* of Math Thematics grade 6 materials. Here students are introduced to proportion and scaling via sculpture.

Exploration The above page from Math Thematics illustrates the importance of using similarity for figures which are not polygons. One possible activity is to have teams of 4 or 5 college students discuss the ways in which similarity for more general figures might be defined. The should also be asked to state what criteria the more general definition should satisfy. In particular, if one generalizes any given definition in mathematics how should one expect the original definition to be related to the new definition?