Calculus Connections

Although the majority of in-service or pre-service middle school mathematics teachers clearly realize the importance of developing a solid understanding of algebra, data analysis, probability and geometry as a necessary preparation for the teaching of mathematics in middle school, the subject of calculus is often viewed in a very different light. For many of these teachers, calculus is seen as a lofty and abstract subject which is "higher" than the mathematics they will teach in the classroom and therefore of little practical value. Many fail to understand the importance of learning calculus as a preparation for the teaching of mathematics in the middle school even though the ideas and principles of calculus lie at the very heart and soul of middle school mathematics. The reason is quite clear: calculus courses do not address these issues!

One of the recommendations outlined in MET for the preparation of middle school mathematics teachers is the development and incorporation of a calculus course that focuses on concepts and applications. In this spirit, we have sought to create a calculus course specifically designed to better prepare middle school mathematics teachers. In this course, we aim to illuminate the connections that exist between the various topics of calculus and the mathematics curricula taught in the middle school.

Philosophy and Structure. Our underlying approach to learning is one of discovery and inquiry. Instead of the usual formula-proof-example approach to calculus, here we initiate each new topic with a classroom discussion or a classroom connection. The classroom discussions are carefully structured sequences of questions designed to lead students to discover the underlying principles of calculus. Detailed answers have been provided for all of the classroom discussions; this gives students the opportunity to independently revisit the ideas discussed in the classroom. The classroom connections are activities from the middle school curricula related to the material to be discussed, and which teachers might someday use in their classrooms. They provide excellent motivation for studying the calculus concepts that are subsequently introduced. The classroom discussions are followed by examples and practice problems. We also give projects and extensions which either focus on applications of the material or provide explorations into new material. The projects and extensions are important tools for further involving students with more in-depth reasoning by sometimes addressing topics that go beyond what is in the text. Each chapter closes with a summary of the topics covered in the chapter and a set of review exercises.

Content. Our selection of topics for this book is based upon a careful study of the existing middle school mathematics curricula. In Chapter 1 we cover sequences and series. Although some calculus textbooks do not include these topics, it is our belief that they should be an integral part of a calculus course offered to future middle school teachers, a view supported by the large number of connections with the middle school curricula included in this chapter. In Chapter 2 we deal with functions – a concept well anchored in the middle school curricula – starting from concrete examples, then fully developing the notion of limit of a function, and closing with a section on continuous functions. Chapter 3 is devoted to differentiation. We start by analyzing average rates of change, move on to instantaneous rates of change and connections with slopes of tangent lines and motion along a straight line, and then prove the main formulas for differentiation. In Chapter 4 we cover optimization, graph sketching, and exponential change. The problems in this chapter underscore the power of calculus when dealing with real-life problems. In Chapter 5, we introduce the notion of integration and discuss basic techniques for evaluating integrals. Integration (or antidifferentiation) is the process that reverses differentiation, and it naturally has a wide variety of practical applications. In this chapter, we discuss how integration can be used to solve various problems related to the study of motion. Chapters 6 and 7 are devoted to exploring applications of integration to various topics from the middle school mathematics curricula: finding areas, lengths, surface areas, and volumes. In these chapters, we begin by computing the area, surface area, and volume of simple figures using Euclidean geometry. Then, building on the results obtained from geometry, we use calculus to produce formulas which hold for more general shapes. Our presentation stresses the interplay between geometry and calculus, and it shows that calculus is a powerful and essential tool for computing areas, lengths, surface areas, and volumes.

Throughout the book there is an emphasis on making a clear distinction between what constitutes a proof and what constitutes an informal explanation. The expectation is that, in the process, students will gradually grow accustomed to a certain standard of rigor that they will, in turn, take back to their classrooms.

To the Instructor. While we envision the classroom discussions as classroom activities to be completed in groups or individually, due to time constraints instructors may choose to assign some of these to be done outside the classroom. Depending on the goals of the course and the mathematical background of the students, instructors might choose to use calculators more extensively, particularly to avoid spending time on computations involving differentiation and integration when the focus is on concepts.